

# K-essence Models and Cosmic Acceleration in Generalized Teleparallel Gravity

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## Abstract

The generalized teleparallel gravity has been suggested to explain the present cosmic acceleration of the universe. In this paper, we take spatially homogenous and anisotropic Bianchi type  $I$  universe in the framework of  $F(T)$  gravity. The behavior of accelerating universe is investigated for three purely kinetic k-essence models. We explore equation of state parameter and deceleration parameter for these k-essence models. It is found that all these models exhibit quintessence behavior of the universe.

**Keywords:**  $F(T)$  gravity; Bianchi type  $I$ ; K-essence.

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## 1 Introduction

The fact that the universe is expanding at every point in space is a difficult concept to grasp. Cosmological observations from cosmic microwave background radiation (CMBR) [1] reveal that most of the energy in our universe is dark which causes gravitational repulsion and hence accelerates expansion of the universe. [2]. The properties of dark energy (DE) can be specified by

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energy density  $\rho$  and pressure  $p$ . These two parameters are responsible for the following three main phases of the universe.

- The first phase is referred to as radiation dominated era which occurred just after creation of the universe. At this stage pressure of the radiation is given by one third of its energy density.
- The next is matter dominated era which came into being when universe was assumed to be 70,000 years old. Its energy density surpassed the energy density of the first phase of the universe until a cosmological constant as a DE candidate was proposed.
- The third era is DE dominated era. About 5 billion years ago this phase dominated in the universe as a whole just after the matter dominated era which was dropped to very low concentration [3]. It is mentioned here that the most recent Wilkinson Microwave Anisotropy Probe (WMAP) observations [2] indicate about 74% of DE in the universe.

By the measurements of CMBR, the WMAP satellite indicates that the universe is very close to the flat and to maintain this flatness, the mass/energy density of the universe must be equal to a certain critical density. The total amount of matter in the universe is estimated only 30% of the critical density. For the remaining 70% of critical density, an additional form of energy is required which is termed as DE. There are two proposed forms to discuss DE: one is the modified theories of gravity and the second is the scalar field models. In this connection, single scalar field models attracted many people. K-essence (k stands for kinetic) model [4] is one of the models which can be described by a single real scalar field  $\phi$  with non-canonical kinetic term responsible for negative pressure. This may be taken as the generalization of canonical scalar fields, e.g., quintessence [5] (a time varying quantity).

The Lagrangian with a non-canonical kinetic term was proposed to discuss the early time acceleration named as k-inflation [6]. Nojiri [7] constructed explicitly k-essence DE model to unify the late-time acceleration and inflation in early universe. Matsumoto and Nojiri [8] reconstructed the scalar quintessence model, tachyon DE model, ghost condensation model as the special cases of k-essence. Armendariz-Picon et al. [9] discussed essential features of k-essence and developed some examples. The solution of these examples lead to two results: one in which cosmic acceleration continues forever

and the other in which acceleration has finite duration. Bose and Majumdar [10] investigated purely kinetic k-essence and a particular k-essence model with potential term. They concluded that such a model could generate basic features of early inflation and DE observational constraints. Yang and Gao [11] introduced purely kinetic k-essence by means of Lagrangian. They plotted evolutions of equation of state (EoS) and speed of sound for particular cases.

Modified theories also play an important role in explaining DE. There exist many modified gravity theories which may naturally unify inflation in early universe and late-time acceleration [12].  $F(T)$  gravity [13] is the modified form of teleparallel equivalent of General Relativity [14]. This theory is formed by using Weitzenböck connection which has no curvature but only torsion and possesses second order set of the field equations.

Myrzakulov [15] proposed some new models of k-essence in the framework of  $F(T)$  gravity. Tsyba et al. [16] investigated purely kinetic k-essence for an explanation of cosmic acceleration. They concluded that for a particular case of scalar field, the modified gravity and purely kinetic k-essence become equivalent. Dent et al. [17] investigated this extended modified gravity at the background and perturbed level and also explored this theory for quintessence scenarios. Karami and Abdolmaleki [18] found that EoS parameter of holographic and new agegraphic  $F(T)$  models always cross the phantom-divide line whereas entropy-corrected model has to experience some conditions on parameters of the model. Most of the above mentioned work has been carried out by using FRW universe for the obvious reasons.

In this paper, we discuss cosmic acceleration by using purely kinetic k-essence and  $F(T)$  gravity with Bianchi type  $I$  universe. The format of the paper is as follows: In the next section, we present some basics of k-essence model and  $F(T)$  gravity theory. In section **3**, the field equations are formulated. Section **4** is devoted to study some k-essence models and also discuss cosmic acceleration. In the last section, some concluding remarks are given.

## 2 Preliminaries

In this section, we provide basic concepts of k-essence as well as  $F(T)$  gravity.

## 2.1 K-essence Formalism

There are some scalar fields with non-canonical kinetic terms in particle physics. The k-essence models are described by a single scalar field and a kinetic term. The general k-essence action [6] is of the form

$$S = \int d^4x \sqrt{-g} L(\phi, X), \quad (1)$$

where  $\phi$  is the scalar field,  $g = \det(g_{\mu\nu})$  and  $X$  is the dimensionless kinetic energy term defined by

$$X = \frac{1}{2} g^{\mu\nu} \phi_\mu \phi_\nu, \quad \mu, \nu = 0, 1, 2, 3, \quad (2)$$

where  $\phi_\mu = \frac{\partial\phi}{\partial x^\mu} = \partial_\mu \phi$ . It is mentioned here that a Lagrangian can be a function of any scalar field  $\phi$  and  $X$ , i.e.,  $L = K(\phi, X)$ . Here we consider one of the simplest possible k-essence model, termed as, purely kinetic k-essence with action

$$S = \int d^4x \sqrt{-g} L(X), \quad L = K(X). \quad (3)$$

The energy-momentum tensor of k-essence is obtained by varying this action with respect to the metric tensor [6]

$$T_{\mu\nu} = K_X \phi_\mu \phi_\nu - K g_{\mu\nu}, \quad K_X = \frac{dK}{dX}. \quad (4)$$

The energy-momentum tensor of perfect fluid is

$$T_{\mu\nu} = (\rho_k + p_k) u_\mu u_\nu - p_k g_{\mu\nu}. \quad (5)$$

Here,  $\rho_k$ ,  $p_k$  are the energy density and pressure of k-essence respectively and  $u_\mu$  is the four-velocity defined by

$$u_\mu = \eta \frac{\phi_\mu}{\sqrt{2X}}, \quad (6)$$

where  $\eta = \pm 1$  according to the sign of  $\dot{\phi}$  positive or negative respectively. We assume that  $\partial_\mu \phi$  is timelike and smooth on interesting scales [5]. Thus we can associate the energy-momentum tensors of k-essence and perfect fluid.

For  $\mu = 0 = \nu$  and  $\mu = 1 = \nu$  in Eqs.(4) and (5), we obtain the following expressions for k-essence energy density and pressure respectively

$$\rho_k = 2XK_X - K, \quad p_k = K. \quad (7)$$

This yields the following k-essence EoS parameter

$$\omega_k = \frac{p_k}{\rho_k} = \frac{K}{2XK_X - K}. \quad (8)$$

## 2.2 $F(T)$ Theory of Gravity

Here, we introduce briefly the teleparallel theory of gravity and its generalization to  $F(T)$  gravity. In teleparallel action, the torsion scalar  $T$  is used as the Lagrangian density while in modified teleparallel gravity, it is promoted to a function of  $T$ . Thus the action for  $F(T)$  gravity [18] is

$$S = \frac{1}{2\kappa^2} \int d^4x [eF(T) + L_m], \quad (9)$$

where  $e = \sqrt{-g}$ ,  $\kappa^2 = 8\pi G$ ,  $G$  is the gravitational constant and  $F(T)$  is the general differentiable function of  $T$ . Also,  $L_m$  is the Lagrangian density of matter inside the universe. The torsion scalar is given as

$$T = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}, \quad (10)$$

where  $S_\rho{}^{\mu\nu}$  and torsion tensor  $T^\rho{}_{\mu\nu}$  are defined as follows

$$S_\rho{}^{\mu\nu} = \frac{1}{2} (K^{\mu\nu}{}_\rho + \delta_\rho^\mu T^{\theta\nu}{}_\theta - \delta_\rho^\nu T^{\theta\mu}{}_\theta), \quad (11)$$

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\nu\mu} - \Gamma^\lambda{}_{\mu\nu} = h_i^\lambda (\partial_\mu h_\nu^i - \partial_\nu h_\mu^i). \quad (12)$$

Here  $h_\mu^i$  are tetrad components which form an orthonormal basis for the tangent space at each point  $x^\mu$  of the manifold. Each vector  $h_i$  can be identified by its components  $h_\mu^i$  such that  $h_i = h_\mu^i \partial_\mu$ , where index  $i$  runs over 0, 1, 2, 3 denote the tangent space while  $\mu = 0, 1, 2, 3$  represent the coordinate indices on the manifold. These tetrad related to the metric tensor  $g_{\mu\nu}$  by the following relation

$$g_{\mu\nu} = \eta_{ij} h_\mu^i h_\nu^j, \quad (13)$$

where  $\eta_{ij} = \text{diag}(1, -1, -1, -1)$  is the Minkowski metric for the tangent space satisfying the following properties

$$h_\mu^i h_j^\mu = \delta_j^i, \quad h_\mu^i h_i^\nu = \delta_\mu^\nu. \quad (14)$$

The contorsion tensor,  $K_\rho^{\mu\nu}$  is equal to the difference between Weitzenböck and Levi-Civita connection defined by

$$K^{\mu\nu}{}_\rho = -\frac{1}{2}(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}). \quad (15)$$

The variation of Eq.(9) with respect to tetrad  $h_\mu^i$  leads to the following field equations

$$[e^{-1}\partial_\mu(eS_i{}^{\mu\nu}) - h_i^\lambda T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu}]F_T + S_i{}^{\mu\nu}\partial_\mu(T)F_{TT} + \frac{1}{4}h_i^\nu F = \frac{1}{2}\kappa^2 h_i^\rho T_\rho^\nu, \quad (16)$$

where  $F_T = \frac{dF}{dT}$ ,  $F_{TT} = \frac{d^2F}{dT^2}$ ,  $S_i{}^{\mu\nu} = h_i^\rho S_\rho{}^{\mu\nu}$  with antisymmetric property. The energy-momentum tensor  $T_{\mu\nu}$  is given as

$$T_\nu^\mu = \text{diag}(\rho_m, -p_m, -p_m, -p_m), \quad (17)$$

where  $\rho_m$  and  $p_m$  denote the usual density and pressure of matter inside the universe.

### 3 Bianchi I Universe and the Field Equations

The assumption of isotropy does not predict early epoch of the big bang as the universe does not maintain its isotropic behavior at very small scales. In order to get a realistic model which represents an expanding, homogenous and anisotropic universe, we use Bianchi type I universe model which is a generalization of FRW metric. Here we study evolution of the universe in the presence of DE k-essence models. The line element of Bianchi type I spacetime is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \quad (18)$$

where the scale factors  $A$ ,  $B$  and  $C$  are functions of cosmic time  $t$  only. Using Eqs.(13) and (18), we obtain the tetrad components as follows [19]

$$h_\mu^i = \text{diag}(1, A, B, C). \quad (19)$$

Using Eqs.(11) and (12) in (10) along with (18), we get

$$T = -2 \left( \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} \right). \quad (20)$$

The field equations (16) for  $i = 0 = \nu$  and  $i = 1 = \nu$  are given by

$$F - 4 \left( \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} \right) F_T = 2\kappa^2 \rho_m, \quad (21)$$

$$\begin{aligned} & 2 \left( \frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} \right) F_T - 4 \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \\ & \times \left[ \left( \frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} \right) \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \left( \frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} \right) \left( \frac{\dot{C}}{C} + \frac{\dot{A}}{A} \right) \right. \\ & \left. + \left( \frac{\ddot{C}}{C} - \frac{\dot{C}^2}{C^2} \right) \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \right] F_{TT} - F = 2\kappa^2 p_m. \end{aligned} \quad (22)$$

The corresponding conservation equation is

$$\dot{\rho}_m + \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) (\rho_m + p_m) = 0. \quad (23)$$

The average scale factor  $R$  and the mean Hubble parameter  $H$  respectively will become

$$R = (ABC)^{1/3}, \quad H = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{\dot{R}}{R}. \quad (24)$$

The deceleration parameter  $q$  is a dimensionless quantity and is used to describe the accelerating universe

$$q = -1 - \frac{\dot{H}}{H^2}. \quad (25)$$

Any cosmological universe represents an accelerating, decelerating or expansion with constant velocity for  $-1 \leq q < 0$ ,  $q > 0$  and  $q = 0$  respectively.

In terms of Hubble parameter and torsion, Eqs.(21)-(23) can be simplified as

$$2\kappa^2\rho_m = 2TF_T + F, \quad (26)$$

$$2\kappa^2 p_m = (6\dot{H} - T + 2J + 2L)F_T + 2M\dot{T}F_{TT} - F, \quad (27)$$

$$0 = \dot{\rho}_m + 3H(\rho_m + p_m), \quad (28)$$

where  $L = \frac{\dot{B}\dot{C}}{BC} - \frac{\ddot{A}}{A}$ ,  $M = \frac{\dot{B}}{B} + \frac{\dot{C}}{C}$  and  $J = \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}$ . Also, Eq.(20) can be written as

$$T = -9H^2 + J. \quad (29)$$

The case  $F(T) = T$  gives the torsion contribution  $\rho_T$ ,  $p_T$  in Eqs.(26) and (27) which reduce to the following forms

$$\rho_m + \rho_T = \frac{3T}{2\kappa^2}, \quad (30)$$

$$p_m + p_T = \frac{1}{\kappa^2} \left( 3\dot{H} - T + J + L \right). \quad (31)$$

If we take  $\rho_T = \rho_k$ ,  $p_T = p_k$ , then these equations become

$$\rho_m + 2XK_X - K = \frac{3T}{2\kappa^2}, \quad (32)$$

$$p_m + K = \frac{1}{\kappa^2} \left( 3\dot{H} - T + J + L \right). \quad (33)$$

For the sake of simplicity, we assume that  $\rho_m = 0 = p_m = \kappa^2 - 1$ . Using these values and inserting the value of  $T$  from Eq.(29) in (32) and (33), it follows that

$$2XK_X - K = \frac{3}{2} (-9H^2 + J), \quad (34)$$

$$K = 3\dot{H} + 9H^2 + L. \quad (35)$$

Equation (2) implies that  $X = \frac{1}{2}\dot{\phi}^2$  whose corresponding scalar function is given by

$$\phi = \int \sqrt{2X} dt + \text{constant}. \quad (36)$$

The continuity equation (5) for k-essence models leads to

$$\dot{\rho}_k + 3H(\rho_k + p_k) = 0. \quad (37)$$



Using Eq.(7) in this equation, we have

$$(K_X + 2XK_{XX})\dot{X} + 6HXK_X = 0 \quad (38)$$

which is the kinetic k-essence field equation.

As a special case [15], we take  $\phi = \phi_0 + \ln R^{\pm\sqrt{18}}$ , where  $\phi_0$  is an arbitrary constant. Consequently, the kinetic term takes the form

$$X = \frac{1}{2}\dot{\phi}^2 = J - T. \quad (39)$$

Inserting this value in Eq.(38), we get

$$2T(\dot{J} - 18H\dot{H})F_{TT} + (\dot{J} - 18H\dot{H} + 6HT)F_T = 0. \quad (40)$$

Also, Eq.(27) can be simplified as

$$2M(\dot{J} - 18H\dot{H})F_{TT} + (6\dot{H} + 9H^2 + J + 2L)F_T - F = 0. \quad (41)$$

It is interesting to mention here that if we use Eq.(40) in (41), it shows the equivalence of  $F(T)$  gravity and k-essence for FRW spacetime [15],[16]. However, these equation do not provide any such relation for Bianchi type  $I$  universe.

## 4 K-essence Models

In this section, we aim to discuss k-essence models in the framework of modified teleparallel gravity. We take three arbitrary k-essence models and evaluate kinetic term and the corresponding scalar field. Also, we formulate EoS parameter and the deceleration parameter for k-essence models. For this purpose, we take the following particular values of the scale factors by using the power law [20]

$$A(t) = (m_1 s_1 t)^{1/m_1}, \quad B(t) = (m_2 s_2 t)^{1/m_2}, \quad C(t) = (m_3 s_3 t)^{1/m_3}, \quad (42)$$

where  $m_i, s_i$  are positive constants and  $i = 1, 2, 3$ . The EoS parameter in Eq.(8) can be written as follows

$$\omega_k = -1 + \frac{2XK_X}{2XK_X - K}. \quad (43)$$

## 4.1 Model I

Consider the following k-essence model [16]

$$K = \sum_{j=0}^M \nu_j(t) y^j, \quad M > 0, \quad (44)$$

where  $y = \tanh t$ . We find the coefficients of  $y$  and the kinetic term of k-essence ( $X$ ) by assuming the following Hubble parameter

$$H = \sum_{j=0}^N \mu_j(t) y^j, \quad N > 0. \quad (45)$$

Notice that we take  $\mu_j(t)$  and  $\nu_j(t)$  as constant quantities throughout. As a particular example, we take  $M = 2$  in Eq.(44) and  $M = 1$  for  $H$ , it follows that

$$K = \nu_0 + \nu_1 y + \nu_2 y^2, \quad H = \mu_0 + \mu_1 y. \quad (46)$$

Inserting this value of  $H$  in Eq.(35), we obtain

$$K = 3\mu_1(3\mu_1 - 1)y^2 + 18\mu_0\mu_1 y + 3\mu_1 + 9\mu_0^2 + L. \quad (47)$$

Using the values of  $K$  and  $H$  in Eq.(34), it follows that

$$X = a_1 e^{\int g_1(t) dt}, \quad (48)$$

where  $a_1$  is an integration constant and

$$\begin{aligned} g_1(t) &= [3\mu_1 + \frac{45}{2}\mu_0^2 + L - \frac{3}{2}J - 9\mu_0\mu_1 y - 3\mu_1(\frac{3}{2}\mu_1 + 1)y^2]^{-1} \\ &\times [2[18\mu_0\mu_1 + 3\mu_1 + 9\mu_0^2 + \dot{L} + 6\mu_1(3\mu_1 - 1)y - 18\mu_0\mu_1 y^2 \\ &- 6\mu_1(3\mu_1 - 1)y^3]. \end{aligned} \quad (49)$$

It is mentioned here that the scalar function  $\phi$  for the model (44) can be obtained by using Eqs.(36) and (48).

The k-essence energy density and pressure in Eq.(7) take the following forms

$$\rho_k = -\frac{27}{2}\mu_1^2 y^2 - 27\mu_0\mu_1 y - \frac{27}{2}\mu_1^2 + \frac{3}{2}J, \quad (50)$$

$$p_k = 3\mu_1(3\mu_1 - 1)y^2 + 18\mu_0\mu_1 y + 3\mu_1 + 9\mu_0^2 + L. \quad (51)$$

Inserting these values in Eq.(43), the EoS parameter becomes

$$\omega_k = -1 - \frac{3\mu_1 - \frac{9}{2}\mu_0^2 + L + \frac{3}{2}J - 9\mu_0\mu_1y - 3\mu_1(\frac{3}{2}\mu_1 + 1)y^2}{\frac{3}{2}(9\mu_0^2 - J) + 27\mu_0\mu_1y + \frac{27}{2}\mu_1^2y^2}. \quad (52)$$

The viability of this model depends on the possible values of the parameters in Eq.(42). The model shows different behavior initially for unequal  $m_i$  but indicates the same behavior at later times for all values of the parameters. Here we discuss the simple case which leads to the isotropic universe. Using Eq.(42) in this equation and assuming  $\mu_i = 1 = m_i = s_i$ , it follows that

$$\omega_k = -1 + \frac{1 - 11/3t^2 + 6 \tanh t + 5 \tanh^2 t}{9 - 3/t^2 + 18 \tanh t + 9 \tanh^2 t}. \quad (53)$$

For late time acceleration, i.e.,  $t \rightarrow \infty$ , this yields

$$\omega_k |_{t \rightarrow \infty} = -\frac{2}{3} \quad (54)$$

which is shown in **Figure 1**. This represents evolution of k-essence EoS parameter  $\omega_k$  as a function of cosmic time. It is obvious that initially ( $t = 0$ ),  $\omega_k = 0.22$  showing that the universe is lying in a region which contained dust fluid as well as radiation. As the time elapses up to  $t = 0.42$ , k-essence EoS parameter confined in the region  $-\infty < \omega_k < +\infty$ , i.e., the universe evolves from physical matter to DE phase. The EoS parameter bears a negative increment in its value and becomes constant at  $t = 0.48$  towards  $t \rightarrow \infty$ . The constant value of EoS parameter is greater than  $-1$  but less than  $-1/3$  which shows a quintessence era [21].

The deceleration parameter (25) for this model takes the form

$$q = -1 + \frac{\mu_1(1 - \tanh^2 t)}{\mu_0^2 + \mu_1^2 \tanh^2 t + 2\mu_0\mu_1 \tanh t} \quad (55)$$

which implies that  $q |_{t \rightarrow \infty} = -1$  indicating an accelerating universe. Its graphical representation is shown in **Figure 2** which indicates that initially at ( $t = 0$ ),  $q = 0$  expressing an expanding universe with a constant velocity. As time passes, its value decreases and converges towards  $-1$ . Notice that the graph exhibits an ever expanding universe as there is not a single positive value of  $q$ .

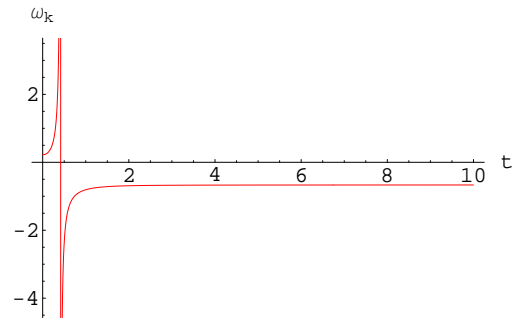


Figure 1: Plot of  $\omega_k$  versus cosmic time  $t$  for model I.

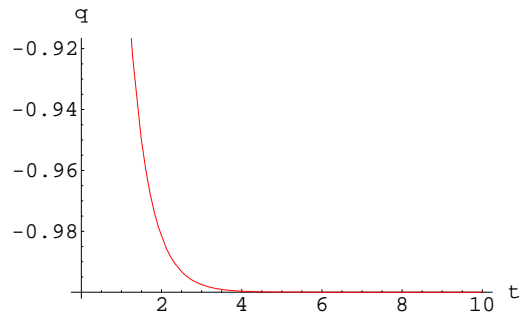


Figure 2: Plot of  $q$  versus cosmic time  $t$  for model I.

## 4.2 Model II

The second k-essence model [15] is given by

$$K = \sum_{j=-m}^n \nu_j(t) e^{jt}, \quad (56)$$

where  $m$  is any positive real number and  $n \leq 0$  real number. For the sake of simplicity, we take  $m = 2$ ,  $n = 0$  and  $\nu_j(t) = \nu_j$  as a constant. Thus

$$K = \nu_{-2} e^{-2t} + \nu_{-1} e^{-t} + \nu_0. \quad (57)$$

We also assume  $H$  as follows

$$H = \mu_{-1} e^{-t} + \mu_0. \quad (58)$$

Inserting this value in Eq.(35), we get

$$K = 9\mu_0^2 + L - 3\mu_{-1}(1 - 6\mu_0)e^{-t} + 9\mu_{-1}^2 e^{-2t}. \quad (59)$$

Using Eqs.(58) and (59) in (34), we obtain kinetic term of k-essence model in the form

$$X = a_2 e^{\int g_2(t) dt}. \quad (60)$$

Here  $a_2$  is an integration constant and  $g_2(t)$  is given by

$$g_2(t) = \frac{2[\dot{L} + 3\mu_{-1}(1 - 6\mu_0)e^{-t} - 18\mu_{-1}^2 e^{-2t}]}{3J/2 - 9/2 + L - 3\mu_{-1}(1 + 3\mu_0)e^{-t} - 9\mu_{-1}^2 e^{-2t}}. \quad (61)$$

The scalar function  $\phi$  is obtained by inserting Eq.(60) in (36).

The energy density and pressure parameters turn out to be

$$\rho_k = -\frac{27}{2}\mu_0^2 + \frac{3}{2}J - 27\mu_{-1}\mu_0 e^{-t} - \frac{27}{2}\mu_{-1}^2 e^{-2t}, \quad (62)$$

$$p_k = 9\mu_0^2 + L - 3\mu_{-1}(1 - 6\mu_0)e^{-t} + 9\mu_{-1}^2 e^{-2t}. \quad (63)$$

Inserting these values in Eq.(43), the EoS parameter takes the form

$$\omega_k = -1 + \frac{9\mu_0^2 - 2L - 3J + 6\mu_{-1}(1 + 3\mu_0)e^{-t} + 9\mu_{-1}^2 e^{-2t}}{27\mu_0^2 - 3J + 54\mu_{-1}\mu_0 e^{-t} + 27\mu_{-1}^2 e^{-2t}}. \quad (64)$$

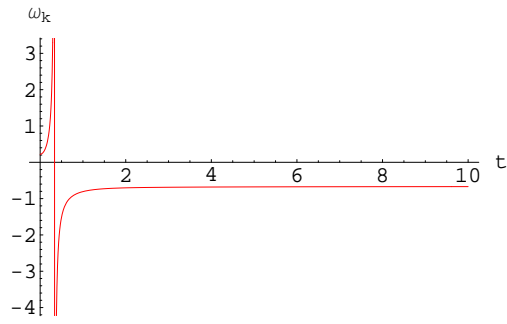


Figure 3: Plot of  $\omega_k$  versus cosmic time for model II.

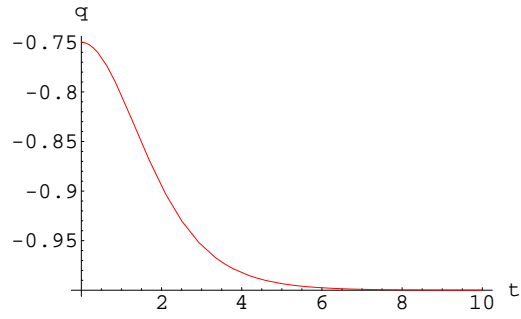


Figure 4: Plot of  $q$  versus cosmic time for model II.

Taking all the constants equal to 1 as in isotropic case and using Eq.(42), it follows that

$$\omega_k = -1 + \frac{9 - 11/t^2 + 24e^{-t} + 9e^{-2t}}{27 - 9/t^2 + 54e^{-t} + 27e^{-2t}}. \quad (65)$$

This shows that as  $t \rightarrow \infty$ ,  $\omega_k > -1$  which represents quintessence region [21] shown in **Figure 3**. This model has the same behavior as that of the first model. However, the EoS parameter is directed to DE era at  $t = 0.34$  and after a short interval of time, it becomes constant, i.e.,  $-0.67$ . The corresponding deceleration parameter is

$$q = -1 + \frac{\mu_{-1}e^{-t}}{\mu_{-1}^2e^{-2t} + \mu_0^2 + 2\mu_{-1}\mu_0e^{-t}}, \quad (66)$$

which gives  $q|_{t \rightarrow \infty} = -1$ . At  $t = 0$ , the value of  $q$  is negative which shows initially an expanding universe. Its value decreases and converges to  $-1$  as  $t \rightarrow \infty$  with the passage of time as shown in **Figure 4**.

### 4.3 Model III

Here we take the following k-essence model [15]

$$K = \sum_{j=-m}^n \nu_j(t)(\ln t)^j, \quad (67)$$

where  $m$ ,  $n$  and  $\nu_j$  are the same as in model II. For  $m = 2$ ,  $n = 0$  and constant  $\nu_j$ 's, this equation becomes

$$K = \nu_{-2}(\ln t)^{-2} + \nu_{-1}(\ln t)^{-1} + \nu_0. \quad (68)$$

Assuming the Hubble parameter in the form

$$H = \mu_{-1}(\ln t)^{-1} + \mu_0. \quad (69)$$

Inserting this value of  $H$  in Eq.(35), it implies that

$$K = L + 9\mu_0^2 + 18\mu_{-1}\mu_0(\ln t)^{-1} + 3\mu_{-1}(3\mu_{-1} - 1/t)(\ln t)^{-2}. \quad (70)$$

The kinetic term  $X$  from (34) and the corresponding scalar function  $\phi$  become

$$X = a_3 e^{\int g_3(t)dt}, \quad \phi = \partial_t^{-1} \sqrt{-2X}, \quad (71)$$

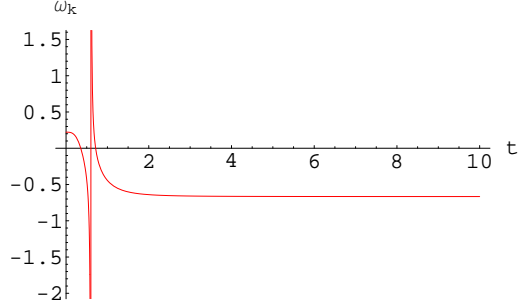


Figure 5: Plot of  $\omega_k$  versus cosmic time for model III.

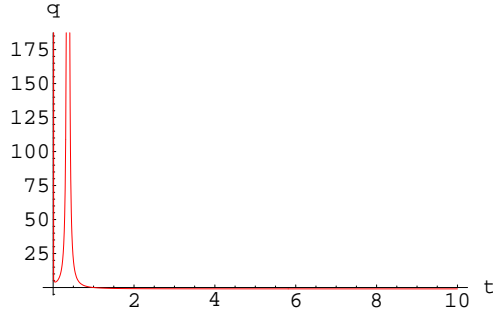


Figure 6: Plot of  $q$  versus cosmic time for model III.

where  $g_3(t)$  is found by using (34)

$$g_3(t) = 2[\dot{L} - 6\mu_{-1}(3\mu_{-1} - 1/t)t^{-1}(\ln t)^{-3} + 3\mu_{-1}(1/t - 6\mu_0)t^{-1}(\ln t)^{-2}] \\ \times [L + 3J/2 - 9\mu_0^2/2 - 9\mu_{-1}\mu_0(\ln t)^{-1} + 3\mu_{-1}(-3\mu_{-1}/2 - 1/t)(\ln t)^{-2}]^{-1}. \quad (72)$$

For this model,  $\rho_k$  and  $p_k$  become

$$\rho_k = \frac{3}{2}J - \frac{27}{2}\mu_0^2 - 27\mu_{-1}\mu_0(\ln t)^{-1} - \frac{27}{2}\mu_{-1}^2(\ln t)^{-2}, \quad (73)$$

$$p_k = L + 9\mu_0^2 + 18\mu_0\mu_{-1}(\ln t)^{-1} + 3\mu_{-1}(3\mu_{-1} - \frac{1}{t})(\ln t)^{-2}. \quad (74)$$



The corresponding EoS parameter takes the form

$$\omega_k = -1 + \frac{2L + 3J - 9\mu_0^2 - 18\mu_{-1}\mu_0(\ln t)^{-1} - 6\mu_{-1}(3\mu_{-1}/2 + 1/t)(\ln t)^{-2}}{3J - 27\mu_0^2 - 54\mu_{-1}\mu_0(\ln t)^{-1} - 27\mu_{-1}^2(\ln t)^{-2}}. \quad (75)$$

To expose the present day nature of the universe, we take  $\mu_i = m_i = s_i = 1$  and using Eq.(42), it follows that

$$\omega_k = -1 + \frac{3 - 11/3t^2 + 6(\ln t)^{-1} + 2(3/2 + 1/t)(\ln t)^{-2}}{9 - 3/t^2 + 18(\ln t)^{-1} + 9(\ln t)^{-2}}. \quad (76)$$

Consequently, the deceleration parameter become

$$q = -1 + \frac{\mu_{-1}}{t(\ln t)^2[\mu_{-1}^2(\ln t)^{-2} + \mu_0^2 + 2\mu_0\mu_{-1}(\ln t)^{-1}]}. \quad (77)$$

This shows that the condition for accelerating universe is satisfied as  $t \rightarrow \infty$ . **Figure 5** (76) shows that  $\omega_k > -1$  as  $t \rightarrow \infty$  and initially the model represents a universe having both properties of matter and radiation. After a very short interval, universe is dominated by DE. It is worthwhile to mention here that at  $t = 0.63$ , the universe changed its phase from DE era to a physical matter dominated era. As compared to the DE phase, the universe stayed in matter dominated era for a short interval of time. Then a decrement in its value is observed, the EoS parameter for k-essence attains negative values and finally converges to a constant value resulting quintessence region [22] of the universe. **Figure 6** (77) represents a decelerating universe at early times as the value of  $q$  is positively increasing or decreasing at that stage. After a long interval of time, the universe intersects the boundary from matter to DE era and takes  $q = -1$ . This corresponds to an accelerating expanding universe.

## 5 Summary

This paper is devoted to study the well-known phenomenon of the universe expansion in the context of  $F(T)$  gravity. For this purpose, we have taken Bianchi type *I* universe and have explored some purely kinetic k-essence models. In physical cosmology and gravity, the DE problem has been investigated in terms of cosmological constant, scalar fields, tachyon, Chaplygin

gas, quintessence and modified gravities etc. Here have used  $F(T)$  gravity and scalar field term whose combination has resulted a quintessence phase.

We have evaluated the EoS parameter and the deceleration parameter for purely kinetic k-essence models. These are shown in terms of graphs versus cosmic time by taking particular values of the metric coefficients. These models describe evolution of the universe from big-bang to present epoch and are summarized as follows

- For model I, the EoS parameter and the deceleration parameter indicate an ever accelerating universe with the passage of time. At  $t = 0.42$ , the universe changes its phase from matter dominated to DE phase resulting quintessence region.
- The model II has the same behavior as that of the model I, however, phase changes at point  $t = 0.38$ . The deceleration parameter initially represents an expanding universe with constant speed and after a short interval, it accelerates with a faster speed.
- For model III,  $\omega_k$  and  $q$  initially show decelerating DE dominated universe which reverses to matter dominated decelerating universe at  $t = 0.64$ . After a slight increment in time, the universe shows accelerating recent epoch of DE issue lying in the quintessence region.

The analysis of models reveal that the present day universe is dominated by DE component which can successfully describe accelerating nature of the universe consistent with the observations [23]. These new forms of models may give the equivalent descriptions of DE to discuss the acceleration of the expanding universe. These models indicate that the present day universe is dominated by DE component. The viability of these models depend on the possible values of parameters. Similarly, we can construct the observational parameters for other new k-essence models induced by modified gravity theory. Finally, it is worthwhile to mention here that all the above results turn out to be the generalization of the already obtained results for the FRW metric [16]. The graphs of models depend upon the values of parameters accordingly. For Bianchi type  $I$ , the k-essence models in the context of generalized teleparallel gravity mark out a quintessence DE phase.

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